A VISIT TO THE ISLAND OF TRUTHTELLERS AND LIARS

Directions: Use logic and truth tables to solve the following exercises.

Faced with engine problems, Ellen Wright put her small plane down on the beach of the Island of Truthtellrs and Liars. Her guidebook offered only the briefest description:

**Island of Truthtellrs and Liars**
The island is fifty-six miles long and up to thirty-three miles wide. A range of inland mountains runs the length of the island. The climate is pleasant through the year. The island has only one city, the capital, Quandary. The most singular feature is the character of the inhabitants. The population is made up entirely of members of two groups, the truth tellers and the liars. The truth tellers utter only true sentences; and the liars, only false sentences. Any inhabitant can recognize whether another person is a truth teller or a liar, but an outsider must judge them by the statements that they make. The actual population is unknown, since no census has proved reliable data.

Ellen decided that her best course was to reach the capital. Walking from the beach, she came to a road that split in two and saw two men working nearby. After Ellen described her arrival on the island, the first man told her, “The capital is in the mountains, or the road to the right goes to the capital.” The second quickly said, “The capital is in the mountains, and the road to the right goes to the capital.” The first looked up and said, “That man is a liar.” The second shrugged his shoulders and added, “If the capital is in the mountains, then the road on the right goes to the capital.”

**Exercise 1**

A. Which way did she go?

\[\begin{array}{c}
1. & p \lor q \quad p \land q \rightarrow \text{false} \\
2. & p \rightarrow q \rightarrow \text{false} \\
\end{array}\]

If 1 is false then 2 is also false
so 1 is true and 2 is a liar

\[\begin{array}{c|c|c|c|c|c}
 & p & q & p \lor q & p \land q & p \rightarrow q \\
\hline
T & T & T & T & F & T \\
T & F & T & T & F & F \\
F & T & T & F & T & F \\
F & F & F & F & T & T \\
\end{array}\]

Answer 1A: The capital is in the mountains and the road to the right doesn't go to the capital. Go to the left.
After an hour walking up the road to the left, Ellen encountered a group of people gathered at what she hoped was a bus stop. She approached three women and asked them whether the road went to the capital and whether this location was a bus stop. She received three different answers: “The road goes to the capital, and the bus stop is not here”; “The road does not go to the capital, and the bus stop is here”; and “The road does not go to the capital, and the bus stop is not here.” Puzzled for a moment, Ellen asked the women whether they were truth tellers or liars. This time, the three answers were all the same: “Two of us are truth tellers, and one is a liar.”

**Exercise 2**

A. How many of the three women were truth tellers?

B. Does the road go to the capital? Is a bus stop at the location where Ellen spoke to the women?

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![Logical Reasoning Diagram]

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**Answer 2A:** they are all liars

**Answer 2B:** road goes to the capital and the bus stop is here
Ellen then noticed signs for buses B1, B2, and B3 and approached another trio. The following conversation took place:
Ellen: Where do the buses go?

_The first._ At least one of B1 and B2 goes to the capital.

_The second._ B1 goes to the capital.

_The third._ B2 and B3 go to the capital.

_The first._ B3 goes to the beach.

_The second._ B2 and B3 go to the beach.

_The third._ B1 goes to the beach.

After some jotting on her guidebook, Ellen stepped onto the right bus and was soon in the capital.

**Exercise 3**

A. Which bus did Ellen take?

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Answer 3A: **B1 goes to the capital**
On reaching the bus station in Quandary, the capital, Ellen noticed three computer terminals. She asked a young woman whether the computers had Internet connections. The woman told her,

"Computer 1 is not connected to the Internet. Ask that man; he is a truth teller." When Ellen approached the man, he told her, "Computer 2 has an Internet connection, but computer 3 does not." A second man, who overheard the conversation, said, "If computer 2 has an Internet connection, then so does computer 1. Computer 3 is not connected to the Internet." With a little more work on the back of her guidebook, Ellen moved to a computer and ordered parts for her plane.

**Exercise 4**

A. Which computer had an Internet connection?

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W1 & M1 must either both be liars or both be truth tellers.

**Answer 4A:** Computer 1